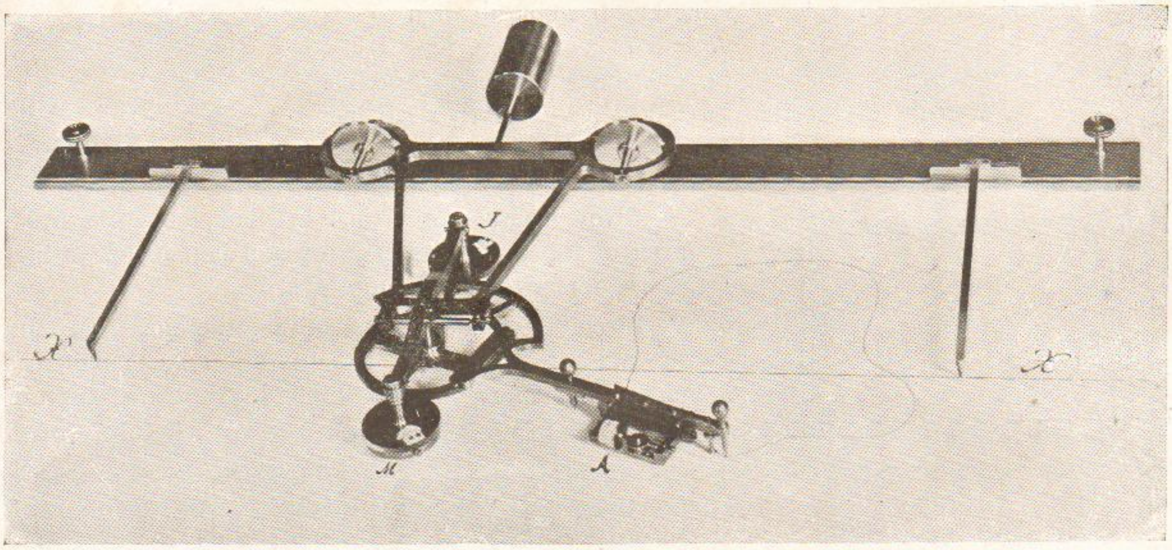
Instructions for using the Integrators.

Nº 1 Integrator.



Nº 1 Integrator measures:

1. the Area.
2. the Moment.
3. the Moment of Inertia.

It does this upon any diagram, by moving a tracing point round its outline.

Dimensions: Longitudinal range 50 ins.
Transverse range 13 ins.
Length of rail 59 ins.
Space between rail and axis of moments 7.5 ins.

Nº 1 Integrator is the pattern mostly used. It is valuable whenever areas, moments and moments of inertia are to be computed. It may also be used for measuring the volume and the position of the centre of gravity of a solid of rotation by its profile.

Nº 1 Integrator is in constant use by Naval Architects.

Directions. Place the rail upon the drawing parallel to the axis of moments **x x** and at such a distance from that line that the points of the two gauges rest **exactly** on the line **x x** while their edges hold in the groove of the rail. Fix the counterweight at the rear of the carriage. The Integrator is shown in use in the figure.

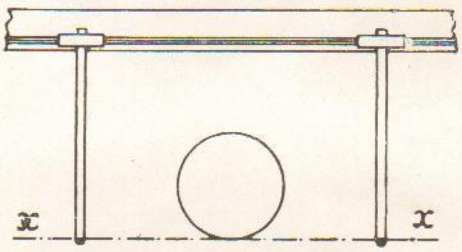
Make a mark on the outline of the diagram to be measured. Set the Nº 1 tracing point on the mark and write down the readings of the three graduated measuring rollers (see on page 7 the heading „Readings"). Move the tracing point carefully along the outline of the diagram from left to right in the direction of the movement of the hands of a watch till it comes back to the starting position. Read off again the measuring rollers and write down the readings under the corresponding first readings. Subtract the first readings from the second and write down the differences on the right of the corresponding readings.

The three figures express then the travel of the respective rollers.
In the following formulae the

travel of roller **A** is denoted by **a**
" " " **M** " " " **m**
" " " **J** " " " **i**

When the fixed No 1 tracing point has been used, then

Area	$A = 0.02\ a$	sq. ins.
Moment	$M = 0.04\ m$	ins. $\times$ sq. ins.
Moment of Inertia	$J = 0.32\ a - 0.1\ i$	sq. ins. $\times$ sq. ins.



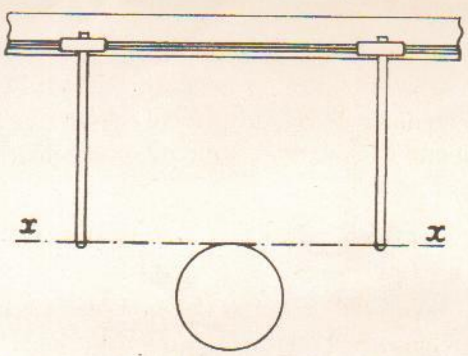
Example:

Given a circle of 4 ins. in diameter. Measure the area of the circle, its moment and its moment of inertia in reference to the tangent **x x**.

Place the rail as shown in the adjoining figure inside the axis of moments. Measure the area, moment and moment of inertia of the circle in the way explained. Thus:

A roller		M roller		J roller	
1832		4495		3721	
2460	628	5123	628	5102	1381
a = 628		m = 628		i = 1381	

Area	$A = 0.02 \times 628 = 12.56$ sq. ins.
Moment	$M = 0.04 \times 628 = 25.12$ ins. $\times$ sq. ins.
Moment of Inertia	$J = 0.32 \times 628 - 0.1 \times 1381 = 62.86$ sq. ins. $\times$ sq. ins.



For another example, place the rail outside the axis of moments and repeat the measurement starting from the same first readings as before. Then:

A roller		M roller		J roller	
1832		4495		3721	
2460	628	3867	— 628	5102	1381
a = 628		m = — 628		i = 1381	

This shows that the **A** and **J** rollers perform the same travel whether the diagram lies inside or outside the axis, but that the **M** roller moves in the opposite direction in each case. The travel in the latter case must therefore be taken into account as a negative quantity.

The travel of the **A** roller is always a forward one. The travel of the **M** roller is forward or backward — positive or negative — according to whether the greater portion of the diagram lies inside or outside the axis of moments. If the travel of the **M** roller turns out negative, the centre of gravity of the diagram lies outside below the axis of moments.

The travel of the **J** roller is in most cases a forward one. It is only negative if the whole diagram lies far off the axis of moments. In such cases the **J** roller may turn backward, and its travel **i** must be taken as negative.

Before or after an exact measurement let the tracing point follow the outline of the diagram roughly, watching the counting discs of the rollers in order to ascertain the direction and the approximate amount of the travel of each roller.

Each measurement claiming reliability for ought to be gone over at least twice.

The tracing arm is provided with a fixed point No 1, and a moveable point No 2 sliding up and down according ~~to the unevenness of the drawing surface.~~ No 2 tracing point is adapted for diagrams with narrow transverse sections. Wherever practicable the diagram should be traced with the moveable point, thereby obtaining a greater travel of the rollers and consequently more accurate results than with the fixed point.

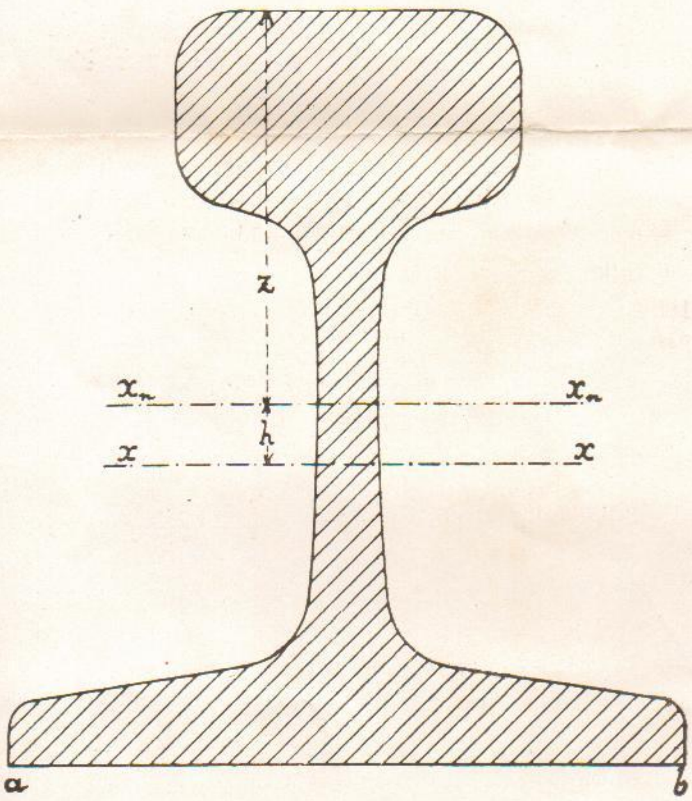
When tracing the diagram by No 2 point, lead the No 1 point with the hand so that No 2 point runs on the outline of the diagram.

When using No 1 point, take off the sliding point.

For No 2 point the following formulae must be used: --

$$A = 0.01\ a \qquad \text{sq. ins.}$$
$$M = 0.01\ m \qquad \text{ins.} \times \text{sq. ins.}$$
$$J = 0.04\ a - \frac{i}{80} \qquad \text{sq. ins.} \times \text{sq. ins.}$$

Example: To determine the resistance of a rail under cross-bending stress. (The diagram below is drawn full size, so that the following results can be obtained again almost exactly by measuring directly from the diagram).



Draw the line **x x** parallel to the foot line **a b** of the rail so that No 2 point of the integrator takes in the whole profile, the line **x x** being chosen as axis of moments.

Set the rail to the line **x x**, measure the area and the moment of the profile by means of No 2 tracing point (the moment of inertia is not wanted now). Thus:

	A roller		M roller	
	9729		1344	
(1)	0164	435	1479	135
(2)	0600	436	1613	134

Means: $a = 435.5 \qquad m = 134.5$

$A = 0.01 \times 435.5 = 4.355$ sq. ins.

$M = 0.01 \times 134.5 = 1.345$ ins. $\times$ sq. ins.

$$h = \frac{M}{A} = \frac{1.345}{4.355} = 0.309 \text{ ins.}$$

h being the height of centre of profile above axis **x x**.

(If **m** were negative, then both **M** and **h** would be negative too, and $x_n\,x_n$ would lie below **x x.**)

Set off the neutral axis $x_n\,x_n$, adjust the rail to the line $x_n\,x_n$ as new axis of moments, and measure again the profile by means of No 2 tracing point taking now readings on all the rollers. Thus:

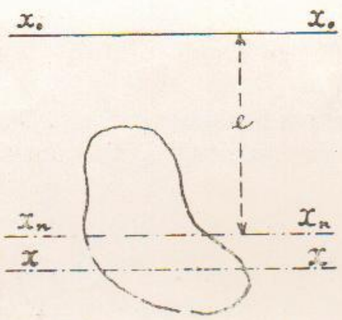
	A roller		M roller		J roller	
	0800		2584		1436	
		435		0		652
	(1) 1235		2584		2088	
		436		— 1		652
	(2) 1671		2583		2740	
means	$a = 435.5$		$m = -0.05$		$i = 652$	

(The measurements of **a** and **m** are to check the foregoing operations).

Moment of Inertia $J_n = 0.04 \times 435.5 - \dfrac{652}{80} = 9.27$ sq. ins. $\times$ sq. ins.

Distance of extreme fibre of rail from neutral axis $z = 2.06$ ins.

Moment of resistance $W = \dfrac{J_n}{z} = \dfrac{9.27}{2.06} = 4.50$ ins. $\times$ sq. ins.



Suppose the diagram is so far off the axis of moments $\mathbf{x_0\ x_0}$ (shown in the adjoining figure), that the tracing point cannot reach the whole outline when the integrator is adjusted to $\mathbf{x_0\ x_0}$.

Then draw another parallel axis **x x** across the diagram so that the whole diagram falls now within the reach of the tracing point when the integrator is set to **x x**. Adjust the integrator to axis **x x** and determine the area and the position of the neutral axis $\mathbf{x_n\ x_n}$ (line through centre of area parallel to $\mathbf{x_0\ x_0}$)

Adjust now the integrator to axis $\mathbf{x_n\ x_n}$ and measure the moment of inertia J_n about axis $\mathbf{x_n\ x_n}$.

If **e** expresses (in inches) the distance between the lines $\mathbf{x_0\ x_0}$ and $\mathbf{x_n\ x_n}$, **A** the area of the diagram, $\mathbf{M_0}$ the moment and $\mathbf{J_0}$ the moment of inertia about the axis $\mathbf{x_0\ x_0}$, then:

$$M_0 = e\ A$$
$$J_0 = J_n + e^2\ A$$

It would be possible, but not advisable, to determine $\mathbf{M_0}$ and J_0 directly from the measurement about axis **x x**.

Large diagrams exceeding the range of the tracing point must be cut into smaller portions. If it is found impossible, to outline the single portions with the tracing point after placing the rail of the Integrator on their axis, in each of the portions a supplementary axis of moments is drawn parallel to the given axis.

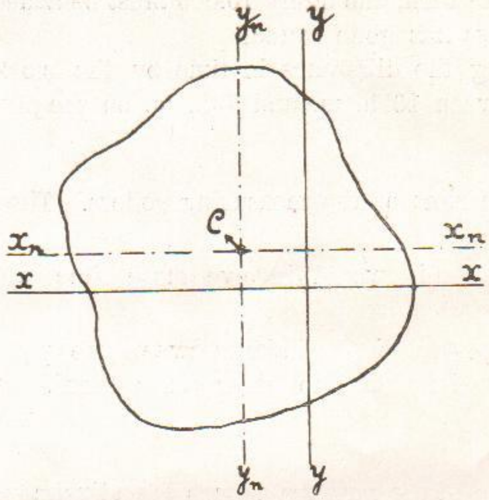
Whereupon determine in each portion of the diagram the neutral axis parallel to the given axis, the moment of inertia about the neutral axis and the area, and then proceed to calculate the moment and the moment of inertia of each portion about the given axis by means of the formulae:

$$M = e\ A \qquad J = J_n + e^2\ A$$

Lastly ascertain the sum of the areas **A** of the single portions, the sum of the moments **M**, and the sum of the moments of inertia **J**.

The above sums will represent the area and the moments of the whole diagram.

Very long but narrow diagrams, as for example the plan of water lines of a ship, cause but little trouble; they may simply be intersected by a set of lines at right angles tho the axis of moments. The sum of the moments of the portions will then make up the moment of the whole diagram.



Position of centre of area. Draw across the diagram any two lines **x x** and **y y** approximately at right angles to one another. Measure the area **A** and the moment $\mathbf{M_x}$ about axis **x x**, then the moment $\mathbf{M_y}$ about axis **y y**. Then $\frac{M_x}{A}$ will be the distance of the line $\mathbf{x_n}$ $\mathbf{x_n}$ from the parallel axis **x x** and $\frac{M_y}{A}$ the distance of $\mathbf{y_n}$ $\mathbf{y_n}$ from **y y**. The point C of intersection of $\mathbf{x_n}$ $\mathbf{x_n}$ and $\mathbf{y_n}$ $\mathbf{y_n}$ will then be the centre of area.

Scales. The foregoing formulae apply to measurements on full size drawings. If the scale of the drawing be 1/n" = 1 ft., the following formulae are to be used:

for No 1 tracing point

$$A = 0.02\ a\ n^2 \qquad \text{sq. ft.}$$
$$M = 0.04\ m\ n^3 \qquad \text{ft.} \times \text{sq. ft.}$$
$$J = (0.32\ a - 0.1\ i)\ n^4 \qquad \text{sq. ft.} \times \text{sq. ft.}$$

and for No 2 tracing point

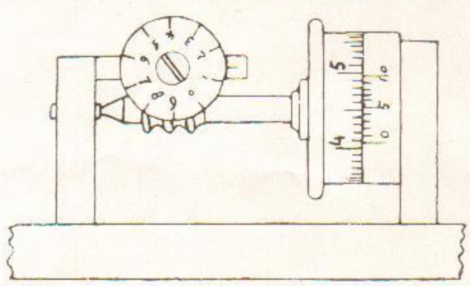
$$A = 0.01\ a\ n^2 \qquad \text{sq. ft.}$$
$$M = 0.01\ m\ n^3 \qquad \text{ft.} \times \text{sq. ft.}$$
$$J = \left\{0.04\ a - \frac{i}{80}\right\} n^4 \qquad \text{sq. ft.} \times \text{sq. ft.}$$

If for example the scale be 1/4" = 1', then

$n = 4 \qquad n^2 = 16 \qquad n^3 = 64 \qquad n^4 = 256.$

Readings. The drum of each roller is divided into 100 parts. The tenths of a part are read on the vernier. The complete turns of the roller are read on the counting disc which advances one line at every such turn. The counting disc performs one revolution at every 10 turns of the roller.

Each complete reading is a figure of 4 digits, the thousands being read on the counting disc, the hundreds and tens on the drum, and the units on the vernier.



The reading of the roller and disc, shown for example in the adjoining figure is 1407.

The zero of the drum and a mark of the disc should simultaneously be opposite the fixed index marks. This is not allways exactly the case in consequence of the imperfection of the worm wheel gear, and this should be taken into account, making readings in the same way as with a watch, when the minute hand points to 12, whilst the hour hand fails to indicate the exact hour.

When taking the second reading after the first measurement, be careful to ascertain whether the motion of the roller has been forward or backward, and whether the zero of the counting disc has passed the fixed index mark as it has an important influence on the reading. If the total travel of the roller has been more than one complete turn, the figure 10.000 must be added to the difference of readings as often as the counting disc has gone round.

The process is as simple, for instance, as calculating the difference in time by the clock between 10 a. m. and 2 p. m. on the same day, or between 10 a. m. and 7 p. m. on the previous day.

General remarks on mechanism. Avoid touching the rims of the measuring rollers. They are liable to be spoiled by rust.

Therefore do not try to set the rollers to zero as this would involve more time and trouble than taking the readings as they stand.

The rollers must rotate very easily and must have very little shake between the pivot centres. The edge of the roller should not touch the vernier. All the other axles should be without back-lash.

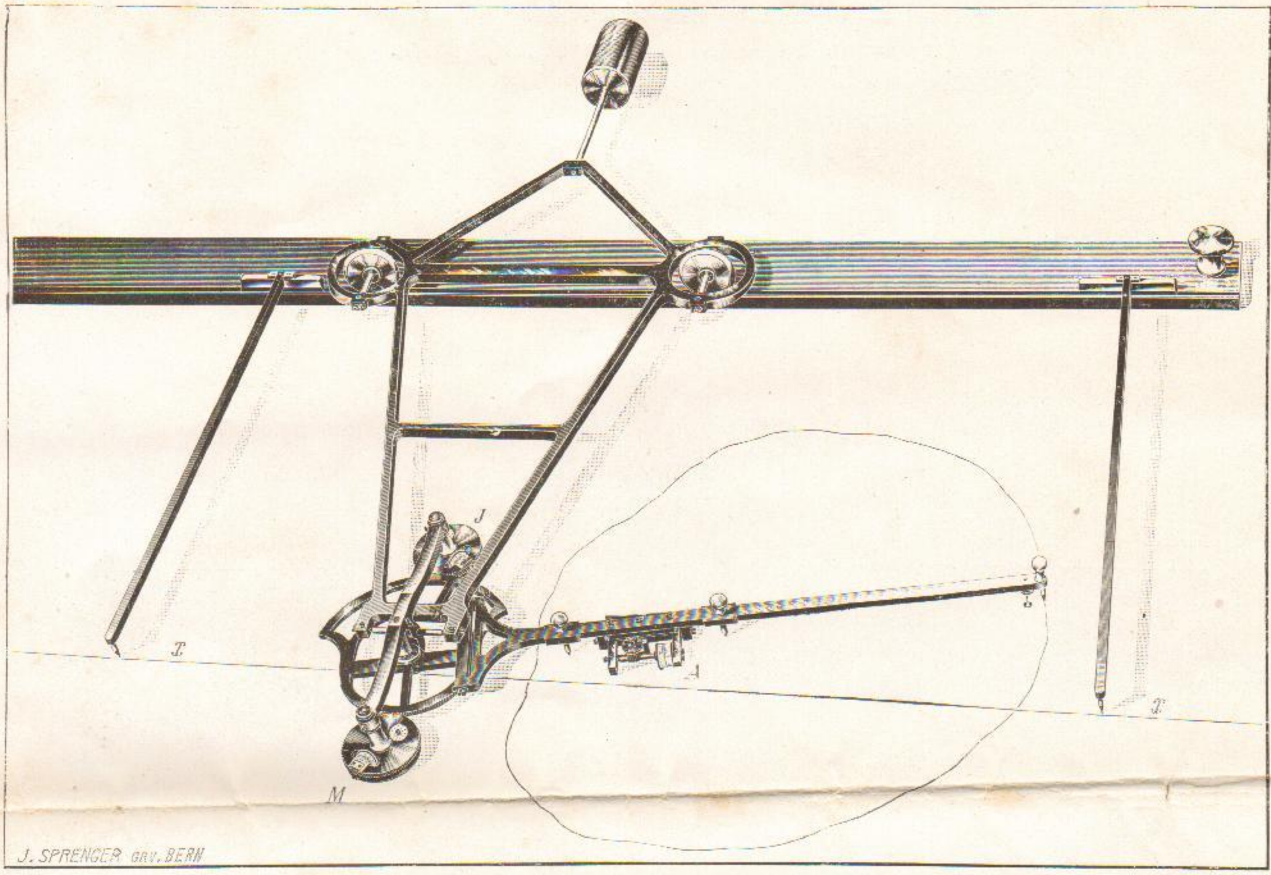
Occasionally oil the pivot centres with fine oil.

Any alterations of the tracing arm or of the wheels of the carriage disturb the adjustment of the integrator and should therefore be avoided.

There is no harm in taking out the measuring rollers, provided that they be carefully handled. The rims of the rollers and the pivot points of the axles are particularly subject to injury.

In case that the adjustments of the integrator have been slightly disturbed it is nevertheless possible to get good results by going over the measurements twice, first in the usual way and secondly by turning the drawing upside down, and then taking the mean results.

Nº 2 Integrator.



Nº 2 Integrator measures:

1. the Area.
2. the Moment.
3. the Moment of Inertia.

It does this upon any diagram.

Dimensions: Longitudinal range 67 ins.
Transverse range 26 ins.
Length of rail 78 ins.
Space between rail and axis of moments 13.5 ins.

Nº 2 Integrator is similar to Nº 1. The only differences are the larger size of Nº 2 and the number of tracing points. Nº 2 is provided with 3 tracing points, viz. one fixed to the tracing arm and two sliding ones.

Nº 2 Integrator may be used for the same purposes as Nº 1, but it has the advantage of a much greater range, which may be useful in Naval Architecture.

Small diagrams can be measured as accurately as by Nº 1 Integrator.

The instructions for Nº 1 Integrator apply equally to Nº 2.

With Nº 2 Integrator the following formulae hold

for Nº 1 tracing point:

$A = 0.04\ a$ sq. ins.

$M = 0.16\ m$ ins. $\times$ sq. ins.

$J = 2.56\ a - 0.8\ i$ sq. ins $\times$ sq. ins.

for N° 2 tracing point:

$$A = 0.02\ a \quad \text{sq. ins.}$$

$$M = 0.04\ m \quad \text{ins.} \times \text{sq. ins.}$$

$$J = 0.32\ a - 0.1\ i \quad \text{sq. ins.} \times \text{sq. ins.}$$

for N° 3 tracing point:

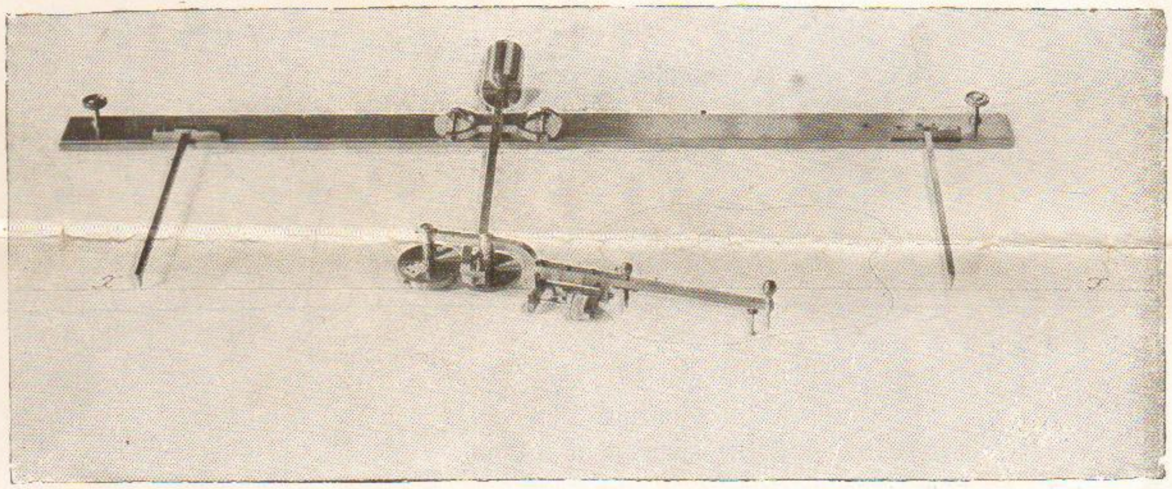
$$A = 0.01\ a \quad \text{sq. ins.}$$

$$M = 0.01\ m \quad \text{ins.} \times \text{sq. ins.}$$

$$J = 0.04\ a - \frac{i}{80} \quad \text{sq. ins.} \times \text{sq. ins.}$$

Always choose that point for tracing a diagram which gives the shortest possible tracing arm for taking in the whole diagram.

N° 3 Integrator.



N° 3 Integrator measures:

1. the Area.
2. the Moment.

It does this upon any diagram.

The moment of Inertia cannot be measured.

Dimensions: Longitudinal range 26 ins.
Transverse range 15 ins.
Length of rail 29 ins.
Space between rail and axis of moments 7.5 ins.

N° 3 Integrator may be used in Naval Architecture for measuring displacement, centre of buoyancy, volume and centre of holds, and for calculations of stability.

In general work, this integrator is valuable whenever areas, moments, centres of gravity of diagrams, volumes of solids of rotation are to be determined.

N° 3 Integrator is the simplest integrating machine, with the exception of the ordinary planimeter. It is a very smooth working and handy instrument.

The instructions for N° 1 Integrator apply equally to N° 3.

With N° 3 Integrator the following formulae hold

for N° 1 tracing point:

$A = 0.02\ a$ sq. ins.

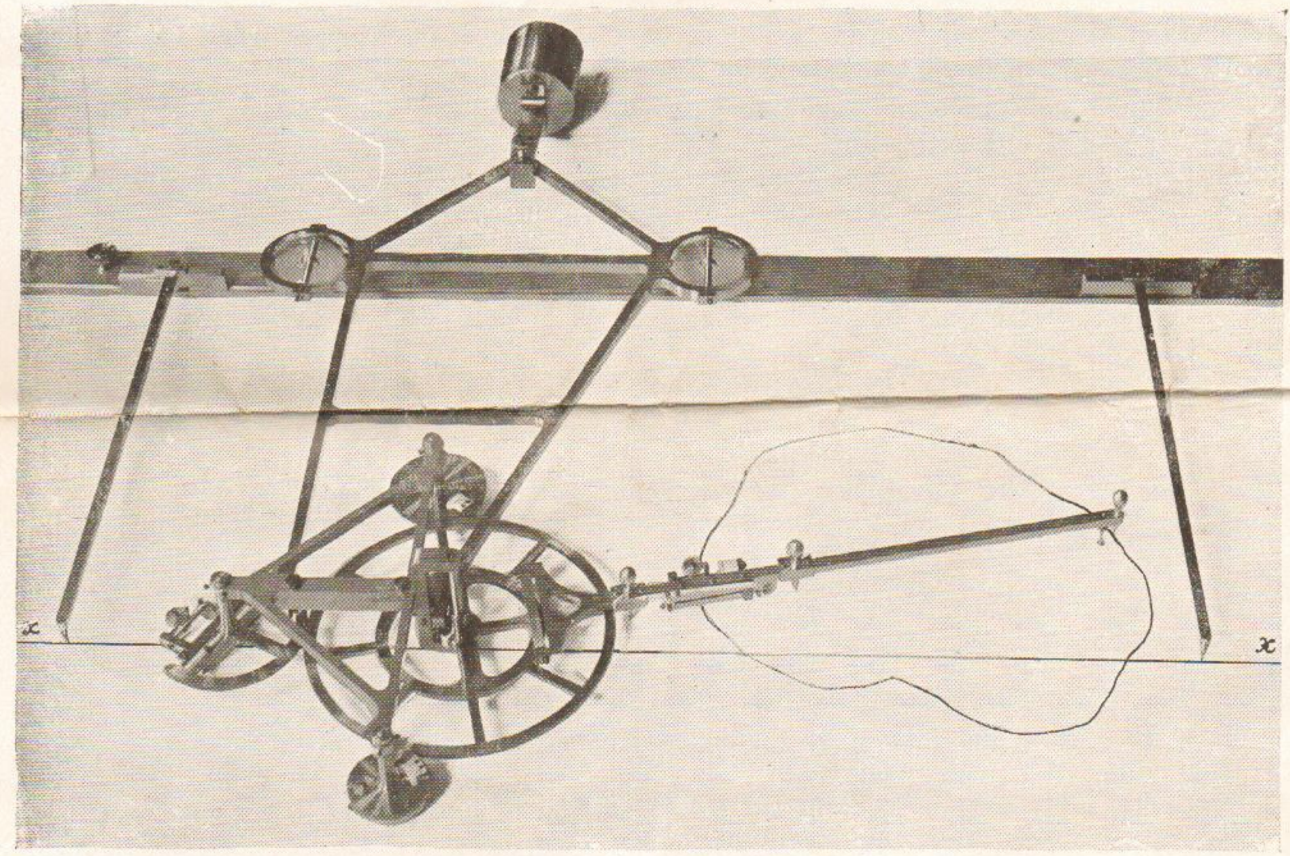
$M = 0.04\ m$ ins. $\times$ sq. ins.

for N° 2 tracing point:

$A = 0.01\ a$ sq. ins.

$M = 0.01\ m$ ins. $\times$ sq. ins.

N° 4 Integrator.



N° 4 Integrator measures:

1. the Area $\int y\ dx = A$
2. the Moment $\frac{1}{2}\int y^2\ dx = M$
3. the Moment of Inertia $\frac{1}{3}\int y^3\ dx = J$
4. the Moment of 4th order $\frac{1}{4}\int y^4\ dx = P$

It does this upon any diagram.

Dimensions: Longitudinal range 67 ins.
Transverse range 24 ins.
Length of rail 78 ins.
Space between rail and axis of moments 12.4 ins.

N° 4 Integrator is chiefly used in *ballistic* investigations. Volume, position of centre of gravity and moment of inertia about any axis of projectiles, guns, armoured turrets, flying wheels, and other solids of rotation can be measured by the profile.

The ranges of N° 4 Integrator, when measuring the moment and moment of inertia about a cross-axis depends on the shape of the profile. A cylinder 47 ins. long and 8 ins. in diameter, for example, is the longest cylinder of 8 ins. in diameter that can be measured, The shorter the cylinder, the larger the diameter may be

N° 4 Integrator is provided with 4 measuring rollers and with 3 tracing points.

Both the revolutions of the M roller and of the P roller and consequently the value of the moment of 4^{th} order may turn out to be negative and this again happens when the greater part of the diagram lies outside below the axis of moments.

With N° 4 Integrator the following formulae hold

for N° 1 tracing point:

$A = 0.04\ a$ sq. ins.

$M = 0.16\ m$ ins. $\times$ sq. ins.

$J = 2.56\ a - 0.8\ i$ sq. ins. $\times$ sq. ins.

$P = 5.12\ (4\ m - p)$ cb. ins. $\times$ sq. ins.

for N° 2 tracing point:

$A = 0.02\ a$

$M = 0.04\ m$

$J = 0.32\ a - 0.1\ i.$

$P = 0.32\ (4\ m - p).$

for N° 3 tracing point:

$A = 0.01\ a$

$M = 0.01\ m$

$$J = 0.04\ a - \frac{i}{80}$$

$P = 0.02\ (4\ m - p).$

a, **m**, **i**, **p** denote the travels of the rollers A, M, J, P respectively.

If the scale of the drawing be $1/n$" = 1 ft., the following formulae are to be used

for N° 1 tracing point:

$A = 0.04\ a\ n^2$ sq. ft.

$M = 0.16\ m\ n^3$ ft. $\times$ sq. ft.

$J = (2.56\ a - 0.8\ i)\ n^4$ sq. ft. $\times$ sq. ft.

$P = 5.12\ (4\ m - p)\ n^5$ cb. ft $\times$ sq. ft.

and so on for N° 2 and N° 3 tracing points.
